

Maximal violation of Bell's inequality and atomic cascade photons

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Abstract

A correlation inequality is derived from local realism and a supplementary assumption. This inequality is violated by a factor of 1.5 in the case of real experiments, whereas previous inequalities such as Clauser-Horne-Shimony-Holt inequality of 1969 and Clauser-Horne inequality of 1974 are violated by a factor of $\sqrt{2}$. Thus the magnitude of violation of this inequality is approximately 20.7% larger than the magnitude of violation of previous inequalities. Moreover, the present inequality can be used to test locality very simply because it requires the measurements of only two detection probabilities. In contrast, Clauser-Horne inequality requires the measurements of five detection probabilities.

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I. Introduction

The Copenhagen interpretation of quantum mechanics is based on the fundamental assumption that the wave function, with its statistical interpretation, provides a complete description of physical reality. This assumption has been the object of severe criticism, most notably by Einstein, who always maintained that the wave function should be supplemented with additional “hidden variables” such that these variables together with the wave function precisely determine the results of individual experiments. In 1965, Bell [1] showed that the premises of locality and realism, as postulated by Einstein, Podolsky, and Rosen (EPR) [2], imply some constraints on the statistics of two spatially separated particles. These constraints, which are collectively known as Bell inequalities, are sometime grossly violated by quantum mechanics. Bell’s theorem therefore is a proof that all realistic interpretation of quantum mechanics must be non-local.

Bell’s original argument, however, can not be experimentally tested because it relies on perfect correlation of the spin of the two particles [3]. Faced with this problem, Clauser-Horne-Shimony-Holt (CHSH) [4], Freedman-Clauser (FC) [5], and Clauser-Horne (CH) [6] derived correlation inequalities for systems which do not achieve 100% correlation, but which do achieve a necessary minimum correlation. Quantum mechanics violates these inequalities by as much as $\sqrt{2}$ (for a detailed discussion of CHSH, FC and CH inequalities, see the review article by Clauser and Shimony [7], especially inequalities 5.3-5.7). An experiment based on CHSH, or FC, or CH inequality utilizes one-channel polarizers in which the dichotomic choice is between the detection of the photon and its lack of detection. A better experiment is one in which a truly binary choice is made between the ordinary and the extraordinary rays. In 1971, Bell [8], and later others [9-11], derived correlation inequalities in which two-channel polarizers are used to test locality. Quantum mechanical probabilities violate these inequalities also by a factor of $\sqrt{2}$. In this paper, we derive a correlation inequality for two-channel polarizer systems and we show that quantum mechanics violates this inequality by a factor of 1.5. Thus the magnitude of violation of the inequality derived in this paper is approximately 20.7% larger than the magnitude of violation of previous inequalities of [4-11]. This result can be of importance for the experimental test of local realism.

II. EPR-Bohm gedankenexperiment

We start by considering Bohm’s [12] version of EPR experiment in which an unstable source emits pairs of photons in a cascade from state $J = 1$ to $J = 0$. The source is viewed by two apparatuses. The first (second)

apparatus consists of a polarizer P_1 (P_2) set at angle \mathbf{a} (\mathbf{b}), and two detectors D_1^\pm (D_2^\pm) put along the ordinary and the extraordinary beams. During a period of time T , the source emits, say, N pairs of photons. Let $N^{\pm\pm}(\mathbf{a}, \mathbf{b})$ be the number of simultaneous counts from detectors D_1^\pm and D_2^\pm , $N^\pm(\mathbf{a})$ the number of counts from detectors D_1^\pm , and $N^\pm(\mathbf{b})$ the number of counts from detectors D_2^\pm . If the time T is sufficiently long, then the ensemble probabilities $p^{\pm\pm}(\mathbf{a}, \mathbf{b})$ are defined as

$$\begin{aligned} p^{\pm\pm}(\mathbf{a}, \mathbf{b}) &= \frac{N^{\pm\pm}(\mathbf{a}, \mathbf{b})}{N}, \\ p^\pm(\mathbf{a}) &= \frac{N^\pm(\mathbf{a})}{N}, \\ p^\pm(\mathbf{b}) &= \frac{N^\pm(\mathbf{b})}{N}. \end{aligned} \tag{1}$$

We consider a particular pair of photons and specify its state with a parameter λ . Following Bell, we do not impose any restriction on the complexity of λ . “It is a matter of indifference in the following whether λ denotes a single variable or a set, or even a set of functions, and whether the variables are discrete or continuous [1].”

The ensemble probabilities in Eq. (1) are defined as

$$\begin{aligned} p^{\pm\pm}(\mathbf{a}, \mathbf{b}) &= \int p(\lambda) p^\pm(\mathbf{a} | \lambda) p^\pm(\mathbf{b} | \lambda, \mathbf{a}), \\ p^\pm(\mathbf{a}) &= \int p(\lambda) p^\pm(\mathbf{a} | \lambda), \\ p^\pm(\mathbf{b}) &= \int p(\lambda) p^\pm(\mathbf{b} | \lambda). \end{aligned} \tag{2}$$

Equations (2) may be stated in physical terms: The ensemble probability for detection of photons by detectors D_1^\pm and D_2^\pm [that is $p^{\pm\pm}(\mathbf{a}, \mathbf{b})$] is equal to the sum or integral of the probability that the emission is in the state λ [that is $p(\lambda)$], times the conditional probability that if the emission is in the state λ , then a count is triggered by the first detector D_1^\pm [that is $p^\pm(\mathbf{a} | \lambda)$], times the conditional probability that if the emission is in the state λ and if the first polarizer is set along axis \mathbf{a} , then a count is triggered from the second detector D_2^\pm [that is $p^\pm(\mathbf{b} | \lambda, \mathbf{a})$]. Similarly the ensemble probability for detection of photons by detector D_1^\pm (D_2^\pm) [that is $p^\pm(\mathbf{a})$ [$p^\pm(\mathbf{b})$]] is equal to the sum or integral of the probability that the photon is in the state λ [that is $p(\lambda)$], times the conditional probability that if the photon is in the state λ , then a count is triggered by detector D_1^\pm (D_2^\pm) [that is $p^\pm(\mathbf{a} | \lambda)$ [$p^\pm(\mathbf{b} | \lambda)$]]. Note that Eqs. (1) and (2) are quite general and follow from the standard rules of probability theory. No assumption has yet been made that is not satisfied by quantum mechanics.

Hereafter, we focus our attention only on those theories that satisfy EPR criterion of locality: “Since at the time of measurement the two systems no longer interact, no real change can take place in the second system in consequence of anything that may be done to first system [2]”. EPR’s criterion of locality can be translated into the following mathematical equation:

$$p^{\pm}(\mathbf{b} \mid \lambda, \mathbf{a}) = p^{\pm}(\mathbf{b} \mid \lambda). \quad (3)$$

Equation (3) is the hall mark of local realism.² It is the most general form of locality that accounts for correlations subject only to the requirement that a count triggered by the second detector does not depend on the orientation of the first polarizer. The assumption of locality, i.e., Eq. (3), is quite natural since the two photons are spatially separated so that the orientation of the first polarizer should not influence the measurement carried out on the second photon.

III. Derivation of Bell’s inequality

We now show that equation (3) leads to validity of an equality that is sometimes grossly violated by the quantum mechanical predictions in the case of real experiments. First we need to prove the following algebraic theorem.

² It is worth noting that there is a difference between Eq. (3) and CH’s criterion of locality. CH write their assumption of locality as

$$p^{+}(\mathbf{a}, \mathbf{b}, \lambda) = p^{+}(\mathbf{a}, \lambda) p^{+}(\mathbf{b}, \lambda).$$

Apparently by $p^{+}(\mathbf{a}, \mathbf{b}, \lambda)$, they mean the conditional probability that if the emission is in state λ , then simultaneous counts are triggered by detectors D_1^{+} and D_2^{+} . However, what they call $p^{+}(\mathbf{a}, \mathbf{b}, \lambda)$ in probability theory is usually called $p^{+}(\mathbf{a}, \mathbf{b} \mid \lambda)$ [note that $p(x, y, z)$ is the joint probability of x, y and z , whereas $p(x, y \mid z)$ is the conditional probability that if z then x and y]. Similarly by $p^{+}(\mathbf{a}, \lambda) [p^{+}(\mathbf{b}, \lambda)]$, CH mean the conditional probability that if the emission is in state λ , then a count is triggered from the detector D_1^{+} (D_2^{+}). Again what they call $p^{+}(\mathbf{a}, \lambda) [p^{+}(\mathbf{b}, \lambda)]$ in probability theory is usually written as $p^{+}(\mathbf{a} \mid \lambda) [p^{+}(\mathbf{b} \mid \lambda)]$ (again note that $p(x, z)$ is the joint probability of x and z , whereas $p(x \mid z)$ is the conditional probability that if z then x). Thus according to standard notation of probability theory, CH criterion of locality may be written as

$$p^{+}(\mathbf{a}, \mathbf{b} \mid \lambda) = p^{+}(\mathbf{a} \mid \lambda) p^{+}(\mathbf{b} \mid \lambda).$$

Now according to Bayes’ theorem,

$$p^{+}(\mathbf{a}, \mathbf{b} \mid \lambda) = p^{+}(\mathbf{a} \mid \lambda) p^{+}(\mathbf{b} \mid \lambda, \mathbf{a}).$$

Substituting the above equation in CH’s criterion of locality, we obtain

$$p^{+}(\mathbf{b} \mid \lambda, \mathbf{a}) = p^{+}(\mathbf{b} \mid \lambda),$$

which for the ordinary equation is the same as Eq. (3).

Theorem: Given ten non-negative real numbers $x_1^+, x_1^-, x_2^+, x_2^-, y_1^+, y_1^-, y_2^+, y_2^-, U$ and V such that $x_1^+, x_1^-, x_2^+, x_2^- \leq U$, and $y_1^+, y_1^-, y_2^+, y_2^- \leq V$, then the following inequality always holds:

$$\begin{aligned} Z &= x_1^+ y_1^+ + x_1^- y_1^- - x_1^+ y_1^- - x_1^- y_1^+ + y_2^+ x_1^+ + y_2^- x_1^- \\ &\quad - y_2^+ x_1^- - y_2^- x_1^+ + y_1^+ x_2^+ + y_1^- x_2^- - y_1^+ x_2^- - y_1^- x_2^+ - 2x_2^+ y_2^+ \\ &\quad - 2x_2^- y_2^- + V x_2^+ + V x_2^- + U y_2^+ + U y_2^- + UV \geq 0. \end{aligned} \quad (4)$$

Proof: Calling $A = y_1^+ - y_1^-$, we write the function Z as

$$\begin{aligned} Z &= x_2^+ (-2y_2^+ + A + V) + x_2^- (-2y_2^- - A + V) \\ &\quad + (x_1^+ - x_1^-) (A + y_2^+ - y_2^-) + U y_2^+ + U y_2^- + UV. \end{aligned} \quad (5)$$

We consider the following eight cases:

$$(1) \text{ First assume } \begin{cases} -2y_2^+ + A + V \geq 0, \\ -2y_2^- - A + V \geq 0, \\ A + y_2^+ - y_2^- \geq 0. \end{cases}$$

The function Z is minimized if $x_2^+ = 0, x_2^- = 0$, and $x_1^+ - x_1^- = -U$. Thus

$$\begin{aligned} Z &\geq -U (A + y_2^+ - y_2^-) + U y_2^+ + U y_2^- + UV \\ &= U (-A + 2y_2^- + V). \end{aligned} \quad (6)$$

Since $V \geq A$ and $y_2^- \geq 0$, $Z \geq 0$.

$$(2) \text{ Next assume } \begin{cases} -2y_2^+ + A + V < 0, \\ -2y_2^- - A + V \geq 0, \\ A + y_2^+ - y_2^- \geq 0. \end{cases}$$

The function Z is minimized if $x_2^+ = U, x_2^- = 0$, and $x_1^+ - x_1^- = -U$. Thus

$$\begin{aligned} Z &\geq U (-2y_2^+ + A + V) - U (A + y_2^+ - y_2^-) + U y_2^+ + U y_2^- + UV \\ &= 2U (V + y_2^- - y_2^+). \end{aligned} \quad (7)$$

Since $V \geq y_2^+$, and $y_2^- \geq 0$, $Z \geq 0$.

$$(3) \text{ Next assume } \begin{cases} -2y_2^+ + A + V \geq 0, \\ -2y_2^- - A + V < 0, \\ A + y_2^+ - y_2^- \geq 0. \end{cases}$$

The function Z is minimized if $x_2^+ = 0, x_2^- = U$, and $x_1^+ - x_1^- = -U$. Thus

$$\begin{aligned} Z &\geq U (-2y_2^- - A + V) - U (A + y_2^+ - y_2^-) + U y_2^+ + U y_2^- + UV \\ &= 2U (V - A). \end{aligned} \quad (8)$$

Since $V \geq A$, $Z \geq 0$.

$$(4) \text{ Next assume } \begin{cases} -2y_2^+ + A + V \geq 0, \\ -2y_2^- - A + V \geq 0, \\ A + y_2^+ - y_2^- < 0. \end{cases}$$

The function Z is minimized if $x_2^+ = 0$, $x_2^- = 0$, and $x_1^+ - x_1^- = U$. Thus

$$\begin{aligned} Z &\geq U(A + y_2^+ - y_2^-) + Uy_2^+ + Uy_2^- + UV \\ &= U(A + 2y_2^+ + V). \end{aligned} \quad (9)$$

Since $V \geq A$ and $y_2^+ \geq 0$, $Z \geq 0$.

$$(5) \text{ Next assume } \begin{cases} -2y_2^+ + A + V < 0, \\ -2y_2^- - A + V < 0, \\ A + y_2^+ - y_2^- \geq 0. \end{cases}$$

The function Z is minimized if $x_2^+ = U$, $x_2^- = U$, and $x_1^+ - x_1^- = -U$. Thus

$$\begin{aligned} Z &\geq U(-2y_2^+ + A + V) + U(-2y_2^- - A + V) - U(A + y_2^+ - y_2^-) \\ &\quad + Uy_2^+ + Uy_2^- + UV \\ &= U(-2y_2^+ - A + 3V). \end{aligned} \quad (10)$$

Since $V \geq A$ and $V \geq y_2^+$, $Z \geq 0$.

$$(6) \text{ Next assume } \begin{cases} -2y_2^+ + A + V < 0, \\ -2y_2^- - A + V \geq 0, \\ A + y_2^+ - y_2^- < 0. \end{cases}$$

The function Z is minimized if $x_2^+ = U$, $x_2^- = 0$, and $x_1^+ - x_1^- = U$. Thus

$$\begin{aligned} Z &\geq U(-2y_2^+ + A + V) + U(A + y_2^+ - y_2^-) + Uy_2^+ + Uy_2^- + UV \\ &= 2U(A + V). \end{aligned} \quad (11)$$

Since $V \geq A$, $Z \geq 0$.

$$(7) \text{ Next assume } \begin{cases} -2y_2^+ + A + V \geq 0, \\ -2y_2^- - A + V < 0, \\ A + y_2^+ - y_2^- < 0. \end{cases}$$

The function Z is minimized if $x_2^+ = 0$, $x_2^- = U$, and $x_1^+ - x_1^- = U$. Thus

$$Z \geq U(-2y_2^- - A + V) + U(A + y_2^+ - y_2^-) + Uy_2^+ + Uy_2^- + UV$$

$$= 2U (y_2^+ - y_2^- + V). \quad (12)$$

Since $V \geq y_2^-$ and $y_2^+ \geq 0$, $Z \geq 0$.

$$(8) \text{ Finally assume } \begin{cases} -2y_2^+ + A + V < 0, \\ -2y_2^- - A + V < 0, \\ A + y_2^+ - y_2^- < 0. \end{cases}$$

The function Z is minimized if $x_2^+ = U$, $x_2^- = U$, and $x_1^+ - x_1^- = U$. Thus

$$\begin{aligned} Z &\geq U(-2y_2^+ + A + V) + U(-2y_2^- - A + V) + U(A + y_2^+ - y_2^-) \\ &\quad + Uy_2^+ + Uy_2^- + UV \\ &= U(-2y_2^- + A + 3V). \end{aligned} \quad (13)$$

Since $V \geq A$ and $V \geq y_2^-$, $Z \geq 0$, and the theorem is proved.

Now let \mathbf{a} (\mathbf{b}) and \mathbf{a}' (\mathbf{b}') be two arbitrary orientation of the first (second) polarizer, and let

$$\begin{aligned} x_1^\pm &= p^\pm(\mathbf{a} | \lambda), & x_2^\pm &= p^\pm(\mathbf{a}' | \lambda), \\ y_1^\pm &= p^\pm(\mathbf{b} | \lambda), & y_2^\pm &= p^\pm(\mathbf{b}' | \lambda). \end{aligned} \quad (14)$$

Obviously for each value of λ , we have

$$\begin{aligned} p^\pm(\mathbf{a} | \lambda) &\leq 1, & p^\pm(\mathbf{a}' | \lambda) &\leq 1, \\ p^\pm(\mathbf{b} | \lambda) &\leq 1, & p^\pm(\mathbf{b}' | \lambda) &\leq 1. \end{aligned} \quad (15)$$

Inequalities (4) and (15) yield

$$\begin{aligned} &p^+(\mathbf{a} | \lambda)p^+(\mathbf{b} | \lambda) + p^-(\mathbf{a} | \lambda)p^-(\mathbf{b} | \lambda) - p^+(\mathbf{a} | \lambda)p^-(\mathbf{b} | \lambda) \\ &- p^-(\mathbf{a} | \lambda)p^+(\mathbf{b} | \lambda) + p^+(\mathbf{b}' | \lambda)p^+(\mathbf{a} | \lambda) + p^-(\mathbf{b}' | \lambda)p^-(\mathbf{a} | \lambda) \\ &- p^+(\mathbf{b}' | \lambda)p^-(\mathbf{a} | \lambda) - p^-(\mathbf{b}' | \lambda)p^+(\mathbf{a} | \lambda) + p^+(\mathbf{b} | \lambda)p^+(\mathbf{a}' | \lambda) \\ &+ p^-(\mathbf{b} | \lambda)p^-(\mathbf{a}' | \lambda) - p^+(\mathbf{b} | \lambda)p^-(\mathbf{a}' | \lambda) - p^-(\mathbf{b} | \lambda)p^+(\mathbf{a}' | \lambda) \\ &- 2p^+(\mathbf{a}' | \lambda)p^+(\mathbf{b}' | \lambda) - 2p^-(\mathbf{a}' | \lambda)p^-(\mathbf{b}' | \lambda) + p^+(\mathbf{a}' | \lambda) \\ &+ p^-(\mathbf{a}' | \lambda) + p^+(\mathbf{b}' | \lambda) + p^-(\mathbf{b}' | \lambda) \geq -1. \end{aligned} \quad (16)$$

Multiplying both sides of (16) by $p(\lambda)$, integrating over λ and using Eqs. (2), we obtain

$$\begin{aligned} &p^{++}(\mathbf{a}, \mathbf{b}) + p^{--}(\mathbf{a}, \mathbf{b}) - p^{+-}(\mathbf{a}, \mathbf{b}) - p^{-+}(\mathbf{a}, \mathbf{b}) + p^{++}(\mathbf{b}', \mathbf{a}) + \\ &p^{--}(\mathbf{b}', \mathbf{a}) - p^{+-}(\mathbf{b}', \mathbf{a}) - p^{-+}(\mathbf{b}', \mathbf{a}) + p^{++}(\mathbf{b}, \mathbf{a}') + \\ &p^{--}(\mathbf{b}, \mathbf{a}') - p^{+-}(\mathbf{b}, \mathbf{a}') - p^{-+}(\mathbf{b}, \mathbf{a}') - 2p^{++}(\mathbf{a}', \mathbf{b}') - \\ &2p^{--}(\mathbf{a}', \mathbf{b}') + p^+(\mathbf{a}') + p^-(\mathbf{a}') + p^+(\mathbf{b}') + p^-(\mathbf{b}') \geq -1. \end{aligned} \quad (17)$$

We now note that the expected value of detection probabilities while polarizers are set along orientations \mathbf{m} and \mathbf{n} , i.e., $E(\mathbf{m}, \mathbf{n})$ is defined as

$$\begin{aligned} E(\mathbf{m}, \mathbf{n}) &= p^{++}(\mathbf{m}, \mathbf{n}) - p^{+-}(\mathbf{m}, \mathbf{n}) \\ &\quad - p^{-+}(\mathbf{m}, \mathbf{n}) + p^{--}(\mathbf{m}, \mathbf{n}). \end{aligned} \quad (18)$$

Using (18), inequality (17) may be written as

$$\begin{aligned} E(\mathbf{a}, \mathbf{b}) + E(\mathbf{b}', \mathbf{a}) + E(\mathbf{b}, \mathbf{a}') - 2p^{++}(\mathbf{a}', \mathbf{b}') - 2p^{--}(\mathbf{a}', \mathbf{b}') \\ + p^+(\mathbf{a}') + p^-(\mathbf{a}') + p^+(\mathbf{b}') + p^-(\mathbf{b}') \geq -1. \end{aligned} \quad (19)$$

All local realistic theories must satisfy inequality (17) [or (19)].

IV. Violation of Bell's inequality in case of ideal experiments

First we consider an atomic cascade experiment in which polarizers and detectors are ideal. Assuming polarizers are set along axes \mathbf{m} and \mathbf{n} where $\theta = |\mathbf{m} - \mathbf{n}|$, the expected values, the single and joint detection probabilities for a pair of photons in a cascade from state $J = 1$ to $J = 0$ are given by

$$\begin{aligned} E(\mathbf{m}, \mathbf{n}) &= E(\theta) = \cos 2\theta, \quad p^+(\mathbf{a}') = p^-(\mathbf{a}') = p^+(\mathbf{b}') = p^-(\mathbf{b}') = \frac{1}{2}, \\ p^{++}(\mathbf{m}, \mathbf{n}) &= p^{++}(\theta) = \frac{\cos^2 \theta}{2}, \quad p^{--}(\mathbf{m}, \mathbf{n}) = p^{--}(\theta) = \frac{\cos^2 \theta}{2}. \end{aligned} \quad (20)$$

Now if we choose the following orientation $(\mathbf{a}, \mathbf{b}) = (\mathbf{b}', \mathbf{a}) = (\mathbf{b}, \mathbf{a}') = 120^\circ$ and $(\mathbf{a}', \mathbf{b}') = 0^\circ$ inequality (19) becomes

$$\begin{aligned} 3E(120^\circ) - 2p^{++}(0^\circ) - 2p^{--}(0^\circ) + \\ p^+(\mathbf{a}') + p^-(\mathbf{a}') + p^+(\mathbf{b}') + p^-(\mathbf{b}') \geq -1. \end{aligned} \quad (21)$$

Using (20), we obtain

$$\begin{aligned} 3 \cos(240^\circ) - 2 \frac{\cos^2(0^\circ)}{2} - 2 \frac{\cos^2(0^\circ)}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} \\ = 3 * (-0.5) - 2 * \frac{1}{2} - 2 * \frac{1}{2} + 2 \geq -1 \end{aligned} \quad (22)$$

or

$$-1.5 \geq -1 \quad (23)$$

which violates inequality (19) by a factor of 1.5 in the case of ideal experiments.

It is worth noting that for ideal polarizers and detectors, inequality (19) is equivalent to CHSH inequality. In an ideal experiment, all emitted photons are analyzed by the detectors and hence the probability that a photon is not collected is zero, i.e.,

$$p^{\pm 0}(\mathbf{m}, \mathbf{n}) = p^{0\pm}(\mathbf{m}, \mathbf{n}) = p^{00}(\mathbf{m}, \mathbf{n}) = 0, \quad (24)$$

where for example $p^{\pm 0}(\mathbf{m}, \mathbf{n})$ [$p^{0\pm}(\mathbf{m}, \mathbf{n})$] is the probability that a count is triggered by the first (second) detector D_1^{\pm} (D_2^{\pm}) and no count is triggered by the second (first) detector D_2^{\pm} (D_1^{\pm}), and $p^{00}(\mathbf{m}, \mathbf{n})$ is the probability that no count is triggered by the detectors. Thus in an ideal experiment,

$$\begin{aligned} p^+(\mathbf{a}') &= p^{++}(\mathbf{a}', \mathbf{b}') + p^{+-}(\mathbf{a}', \mathbf{b}'), \\ p^-(\mathbf{a}') &= p^{-+}(\mathbf{a}', \mathbf{b}') + p^{--}(\mathbf{a}', \mathbf{b}'). \end{aligned} \quad (25)$$

Substituting (25) in (19), we obtain

$$\begin{aligned} E(\mathbf{a}, \mathbf{b}) + E(\mathbf{b}', \mathbf{a}) + E(\mathbf{b}, \mathbf{a}') - p^{++}(\mathbf{a}', \mathbf{b}') + p^{+-}(\mathbf{a}', \mathbf{b}') + \\ p^{-+}(\mathbf{a}', \mathbf{b}') - p^{--}(\mathbf{a}', \mathbf{b}') \geq -1 - p^+(\mathbf{b}') - p^-(\mathbf{b}'). \end{aligned} \quad (26)$$

Since we have assumed detectors and polarizers are ideal, we have $-p^+(\mathbf{b}') - p^-(\mathbf{b}') = -1$. Thus

$$E(\mathbf{a}, \mathbf{b}) + E(\mathbf{b}', \mathbf{a}) + E(\mathbf{b}, \mathbf{a}') - E(\mathbf{a}', \mathbf{b}') \geq -2, \quad (27)$$

which are the same as CHSH inequality.

It is important to emphasize that in the case of ideal experiments, neither the present inequality nor CHSH are necessary: they both immediately reduce to Bell's original inequality of 1965 [1]. To show this we assume \mathbf{a}' and \mathbf{b}' are along the same direction. Since we have assumed polarizers and detectors are ideal, Eqs (20) holds, and we have $E(\mathbf{a}', \mathbf{b}') = 1$. CHSH inequality (27) therefore becomes

$$E(\mathbf{a}, \mathbf{b}) + E(\mathbf{b}', \mathbf{a}) + E(\mathbf{b}, \mathbf{a}') \geq -1. \quad (28)$$

which is the same as Bell's inequality of 1965 [1]. Similar argument shows that for an ideal experiment, inequality (19) reduces to Bell's original inequality [1].

We have therefore shown that for ideal polarizers and detectors where the polarizations of two photons along the same axis are perfectly correlated, Bell's original inequality [1] is sufficient and there is no need for inequality (19) or CHSH inequality. Moreover, we have shown that for the case ideal

experiments (see 24), i. e., for the case where $p^{++}(\mathbf{m}, \mathbf{n}) + p^{+-}(\mathbf{m}, \mathbf{n}) + p^{-+}(\mathbf{m}, \mathbf{n}) + p^{--}(\mathbf{m}, \mathbf{n}) = 1$, inequality (19) is equivalent to CHSH inequality. However, for the case of real experiments where $p^{\pm 0}(\mathbf{m}, \mathbf{n})$, $p^{0\pm}(\mathbf{m}, \mathbf{n})$, and $p^{00}(\mathbf{m}, \mathbf{n})$ are non-zero, i. e., for the case where $p^{++}(\mathbf{m}, \mathbf{n}) + p^{+-}(\mathbf{m}, \mathbf{n}) + p^{-+}(\mathbf{m}, \mathbf{n}) + p^{--}(\mathbf{m}, \mathbf{n}) < 1$, inequality (19) is a distinct and new inequality and is not equivalent to CHSH correlation.

V. Necessity of a supplementary assumption for violation of Bell's inequality in case of real experiments

We now consider the violation of Bell's inequality (19) in the case of real experiments, i.e., in the case of real detectors and polarizers. We consider an atomic cascade experiment in which an atom emits two photons in a cascade from state $J = 1$ to $J = 0$. Since the pair of photons have zero angular momentum, they propagate in the form of spherical wave. Thus the probability $p(\mathbf{d}_1, \mathbf{d}_2)$ of both photons being simultaneously detected by two detectors in the directions \mathbf{d}_1 and \mathbf{d}_2 is [3],[4]

$$p(\mathbf{d}_1, \mathbf{d}_2) = \eta^2 \left(\frac{\Omega}{4\pi} \right)^2 g(\theta, \phi), \quad (29)$$

where η is the quantum efficiency of the detectors, Ω is the solid angle of the detector, $\cos \theta = \mathbf{d}_1 \cdot \mathbf{d}_1$, and angle ϕ is related to Ω by

$$\Omega = 2\pi(1 - \cos \phi). \quad (30)$$

Finally the function $g(\theta, \phi)$ is the angular correlation function and in the special case is given by [4]

$$g(\pi, \phi) = 1 + \frac{1}{8} \cos^2 \phi (1 + \cos \phi)^2. \quad (31)$$

If we insert polarizers in front of the detectors, then the quantum mechanical predictions for single and joint detection probabilities are [3], [4]

$$\begin{aligned} p^+(\mathbf{a}) &= p^-(\mathbf{a}) = \eta \left(\frac{\Omega}{8\pi} \right), & p^+(\mathbf{b}) &= p^-(\mathbf{b}) = \eta \left(\frac{\Omega}{8\pi} \right), \\ p^{++}(\mathbf{a}, \mathbf{b}) &= p^{--}(\mathbf{a}, \mathbf{b}) = \eta^2 \left(\frac{\Omega}{8\pi} \right)^2 g(\theta, \phi) [1 + F(\theta, \phi) \cos 2(\mathbf{a} - \mathbf{b})], \\ p^{+-}(\mathbf{a}, \mathbf{b}) &= p^{-+}(\mathbf{a}, \mathbf{b}) = \eta^2 \left(\frac{\Omega}{8\pi} \right)^2 g(\theta, \phi) [1 - F(\theta, \phi) \cos 2(\mathbf{a} - \mathbf{b})], \end{aligned} \quad (32)$$

where $F(\theta, \phi)$ is the so-called depolarization factor and for the special case $\theta = \pi$ and small ϕ is given by

$$F(\pi, \phi) \approx 1 - \frac{2}{3}(1 - \cos \phi)^2. \quad (33)$$

The function $F(\theta, \phi)$, in general, is very close to 1 (the detailed expression for $F(\theta, \phi)$ is given in [4]).

In the atomic cascade experiments which are feasible with present technology [5,13], because $\frac{\Omega}{4\pi} \ll 1$, only a very small fraction of photons are detected. Thus inequality (17) or (19) can not be used to test the violation of Bell's inequality. It is important to emphasize that a supplementary assumption is required primarily because the solid angle covered by the aperture of the apparatus, Ω , is much less than 4π and not because the efficiency of the detectors, η , is much smaller than 1. In fact in the previous experiments (Ref. 13), the efficiency of detectors were larger than 90%. However, because $\frac{\Omega}{4\pi} \ll 1$, all previous experiments needed supplementary assumptions to test locality.

It is worth nothing that CHSH [4] and CH [6] combined the solid angle covered by the aperture of the apparatus $\frac{\Omega}{4\pi}$ and the efficiency of the detectors η into one term and wrote $\eta_{CHSH} = \eta \frac{\Omega}{4\pi}$. They then referred to η_{CHSH} as efficiency of the detectors (this terminology however, is not usually used in optics. In optics, the efficiency of detector refers to the probability of detection of a photon; it does not refer to the product of the solid angle covered by the detector and the probability of detection of a photon). CHSH and CH then pointed that since $\eta_{CHSH} \ll 1$, a supplementary assumption is required. To clarify CHSH and CH argument, it should be emphasized that a supplementary assumption is needed mainly because $\frac{\Omega}{4\pi} \ll 1$, not because $\eta \ll 1$.

We now state a supplementary assumption and we show that this assumption is sufficient to make these experiments (where $\frac{\Omega}{4\pi} \ll 1$) applicable as a test of local theories. The supplementary assumption is: For every emission λ , the detection probability by detector D^+ (or D^-) is *less than or equal* to the sum of detection probabilities by detectors D^+ and D^- when the polarizer is set along any *arbitrary* axis. If we let \mathbf{r} be an *arbitrary* direction of the first or second polarizer, then the above supplementary assumption may be translated into the following inequalities

$$\begin{aligned} p^+(\mathbf{a} | \lambda) &\leq p^+(\mathbf{r} | \lambda) + p^-(\mathbf{r} | \lambda), & p^-(\mathbf{a} | \lambda) &\leq p^+(\mathbf{r} | \lambda) + p^-(\mathbf{r} | \lambda), \\ p^+(\mathbf{a}' | \lambda) &\leq p^+(\mathbf{r} | \lambda) + p^-(\mathbf{r} | \lambda), & p^-(\mathbf{a}' | \lambda) &\leq p^+(\mathbf{r} | \lambda) + p^-(\mathbf{r} | \lambda), \\ p^+(\mathbf{b} | \lambda) &\leq p^+(\mathbf{r} | \lambda) + p^-(\mathbf{r} | \lambda), & p^-(\mathbf{b} | \lambda) &\leq p^+(\mathbf{r} | \lambda) + p^-(\mathbf{r} | \lambda), \\ p^+(\mathbf{b}' | \lambda) &\leq p^+(\mathbf{r} | \lambda) + p^-(\mathbf{r} | \lambda), & p^-(\mathbf{b}' | \lambda) &\leq p^+(\mathbf{r} | \lambda) + p^-(\mathbf{r} | \lambda). \end{aligned} \quad (34)$$

This supplementary assumption is obviously valid for an ensemble of photons. The sum of detection probability by detector D^+ and D^- for an ensemble of photons when the polarizer is set along any *arbitrary* axis \mathbf{v}

$$p^+(\mathbf{v}) + p^-(\mathbf{v}) = \eta \left(\frac{\Omega}{4\pi} \right), \quad (35)$$

whereas

$$p^+(\mathbf{v}) = p^-(\mathbf{v}) = \eta \left(\frac{\Omega}{8\pi} \right). \quad (36)$$

The supplementary assumption requires that the corresponding probabilities be valid for each λ .

It is worth noting that the present supplementary assumption is weaker than Garuccio-Rapisarda (GR) [9] assumption, that is, an experiment based on the present supplementary assumption refutes a larger family of hidden variable theories than an experiment based on GR assumption. The GR assumption is

$$p^+(\mathbf{a} | \lambda) + p^-(\mathbf{a} | \lambda) = p^+(\mathbf{r} | \lambda) + p^-(\mathbf{r} | \lambda) \quad (37)$$

It is easy to show that GR assumption implies the assumption of this paper. First note that

$$p^+(\mathbf{a} | \lambda) \leq p^+(\mathbf{a} | \lambda) + p^-(\mathbf{a} | \lambda), \quad p^-(\mathbf{a} | \lambda) \leq p^+(\mathbf{a} | \lambda) + p^-(\mathbf{a} | \lambda). \quad (38)$$

Now using GR assumption (37), we can immediately conclude that

$$p^+(\mathbf{a} | \lambda) \leq p^+(\mathbf{r} | \lambda) + p^-(\mathbf{r} | \lambda), \quad p^-(\mathbf{a} | \lambda) \leq p^+(\mathbf{r} | \lambda) + p^-(\mathbf{r} | \lambda), \quad (39)$$

which is the same as (34). Thus an experiment which refutes the hidden variable theories which are consistent with GR assumption also refutes the hidden variable theories which are consistent with the present assumption. The reverse however is not true. An experiment based on the present supplementary assumption refutes a larger family of hidden variable theories than an experiment based on GR assumption.

Now using relations (4), (14) and (34), and applying the same argument that led to inequality (17), we obtain the following inequality

$$\begin{aligned} & \left[p^{++}(\mathbf{a}, \mathbf{b}) + p^{--}(\mathbf{a}, \mathbf{b}) - p^{+-}(\mathbf{a}, \mathbf{b}) - p^{-+}(\mathbf{a}, \mathbf{b}) + p^{++}(\mathbf{b}', \mathbf{a}) + p^{--}(\mathbf{b}', \mathbf{a}) \right. \\ & - p^{+-}(\mathbf{b}', \mathbf{a}) - p^{-+}(\mathbf{b}', \mathbf{a}) + p^{++}(\mathbf{b}, \mathbf{a}') + p^{--}(\mathbf{b}, \mathbf{a}') - p^{+-}(\mathbf{b}, \mathbf{a}') \\ & - p^{-+}(\mathbf{b}, \mathbf{a}') - 2p^{++}(\mathbf{a}', \mathbf{b}') - 2p^{--}(\mathbf{a}', \mathbf{b}') + p^{++}(\mathbf{a}', \mathbf{r}) + p^{+-}(\mathbf{a}', \mathbf{r}) \\ & + p^{-+}(\mathbf{a}', \mathbf{r}) + p^{--}(\mathbf{a}', \mathbf{r}) + p^{++}(\mathbf{r}, \mathbf{b}') + p^{+-}(\mathbf{r}, \mathbf{b}') + p^{-+}(\mathbf{r}, \mathbf{b}') \\ & \left. + p^{--}(\mathbf{r}, \mathbf{b}') \right] / \left[p^{++}(\mathbf{r}, \mathbf{r}) + p^{+-}(\mathbf{r}, \mathbf{r}) + p^{-+}(\mathbf{r}, \mathbf{r}) + p^{--}(\mathbf{r}, \mathbf{r}) \right] \geq -1. \quad (40) \end{aligned}$$

or

$$\begin{aligned} & \left[E(\mathbf{a}, \mathbf{b}) + E(\mathbf{b}', \mathbf{a}) + E(\mathbf{b}, \mathbf{a}') - 2p^{++}(\mathbf{a}', \mathbf{b}') - 2p^{--}(\mathbf{a}', \mathbf{b}') + p^{++}(\mathbf{a}', \mathbf{r}) \right. \\ & + p^{+-}(\mathbf{a}', \mathbf{r}) + p^{-+}(\mathbf{a}', \mathbf{r}) + p^{--}(\mathbf{a}', \mathbf{r}) + p^{++}(\mathbf{r}, \mathbf{b}') + p^{+-}(\mathbf{r}, \mathbf{b}') + p^{-+}(\mathbf{r}, \mathbf{b}') \\ & \left. + p^{--}(\mathbf{r}, \mathbf{b}') \right] / \left[p^{++}(\mathbf{r}, \mathbf{r}) + p^{+-}(\mathbf{r}, \mathbf{r}) + p^{-+}(\mathbf{r}, \mathbf{r}) + p^{--}(\mathbf{r}, \mathbf{r}) \right] \geq -1. \quad (41) \end{aligned}$$

Note that in inequality (40) [or (41)], the number of emissions N from the source is eliminated from the ratio. Using (32), it can easily be seen that quantum mechanics violates inequality (40) [or (41)] in case of real experiments where the solid angle covered by the aperture of the apparatus, Ω , is much less than 4π . In particular, the magnitude of violation is maximized if the following orientations are chosen $(\mathbf{a}, \mathbf{b}) = (\mathbf{b}', \mathbf{a}) = (\mathbf{b}, \mathbf{a}') = 120^\circ$ and $(\mathbf{a}', \mathbf{b}') = (\mathbf{a}', \mathbf{r}) = (\mathbf{r}, \mathbf{b}') = 0^\circ$.

Inequality (41) may be considerably simplified if we invoke some of the symmetries that are exhibited in atomic-cascade photon experiments. For a pair of photons in cascade from state $J = 1$ to $J = 0$, the quantum mechanical detection probabilities $p_{QM}^{\pm\pm}$ and expected value E_{QM} exhibit the following symmetry

$$p_{QM}^{\pm\pm}(\mathbf{a}, \mathbf{b}) = p_{QM}^{\pm\pm}(|\mathbf{a} - \mathbf{b}|), \quad E_{QM}(\mathbf{a}, \mathbf{b}) = E_{QM}(|\mathbf{a} - \mathbf{b}|). \quad (42)$$

We assume that the local theories also exhibit the same symmetry

$$p^{\pm\pm}(\mathbf{a}, \mathbf{b}) = p^{\pm\pm}(|\mathbf{a} - \mathbf{b}|), \quad E(\mathbf{a}, \mathbf{b}) = E(|\mathbf{a} - \mathbf{b}|). \quad (43)$$

Note that there is no harm in assuming Eqs. (43) since they are subject to experimental test (CHSH [4], FC [5], and CH [6] made the same assumptions). Using the above symmetry, inequality (41) may be written as

$$\begin{aligned} & \left[E(|\mathbf{a} - \mathbf{b}|) + E(|\mathbf{b} - \mathbf{a}'|) + E(|\mathbf{b}' - \mathbf{a}|) - 2p^{++}(|\mathbf{a}' - \mathbf{b}'|) - 2p^{--}(|\mathbf{a}' - \mathbf{b}'|) \right. \\ & + p^{++}(|\mathbf{a}' - \mathbf{r}|) + p^{+-}(|\mathbf{a}' - \mathbf{r}|) + p^{-+}(|\mathbf{a}' - \mathbf{r}|) + p^{--}(|\mathbf{a}' - \mathbf{r}|) \\ & + p^{++}(|\mathbf{r} - \mathbf{b}'|) + p^{+-}(|\mathbf{r} - \mathbf{b}'|) + p^{-+}(|\mathbf{r} - \mathbf{b}'|) + p^{--}(|\mathbf{r} - \mathbf{b}'|) \left. \right] / \\ & \left[p^{++}(0^\circ) + p^{+-}(0^\circ) + p^{-+}(0^\circ) + p^{--}(0^\circ) \right] \geq -1. \quad (44) \end{aligned}$$

We now take \mathbf{a}' and \mathbf{b}' to be along direction \mathbf{r} , and we take \mathbf{a} , \mathbf{b} , and \mathbf{a}' to be three coplanar axes, each making 120° with the other two, that is we choose the the following orientations, $|\mathbf{a} - \mathbf{b}| = |\mathbf{b}' - \mathbf{a}| = |\mathbf{b} - \mathbf{a}'| = 120^\circ$ and $|\mathbf{a}' - \mathbf{b}'| = |\mathbf{a}' - \mathbf{r}| = |\mathbf{r} - \mathbf{b}'| = 0^\circ$. Furthermore if we define K as

$$K = p^{++}(0^\circ) + p^{+-}(0^\circ) + p^{-+}(0^\circ) + p^{--}(0^\circ) \quad (45)$$

then the above inequality is simplified to

$$\frac{3E(120^\circ) + 2p^{+-}(0^\circ) + 2p^{-+}(0^\circ)}{K} \geq -1. \quad (46)$$

Using the quantum mechanical probabilities³ [i.e., Eqs. (38)], inequality (46) becomes $-1.5 \geq -1$ in the case of real experiments (here for simplicity, we have assumed $F(\theta, \phi) = 1$; this is a good approximation even in the case of real experiments. In actual experiments where the solid angle of detectors ϕ is usually less than $\pi/6$, from (33) it can be seen that $F(\theta, \pi/6) \approx 0.99$). Quantum mechanics therefore violates inequality (44) (or (46)) by a factor of 1.5, whereas it violates CH (or CHSH) inequality by a factor of $\sqrt{2}$. Thus the magnitude of violation of inequality (46) is approximately 20.7% larger than the magnitude of violation of the previous inequalities [4-10]. Inequality (46) can be used to test locality considerably more simply than CH or CHSH inequality.

CH inequality may be written as

$$\frac{3p(\phi) - p(3\phi) - p(\mathbf{a}', \infty) - p(\infty, \mathbf{b})}{p(\infty, \infty)} \leq 0. \quad (47)$$

The above inequality requires the measurements of five detection probabilities:

- (1) The measurement of detection probability with both polarizers set along the 22.5° axis [that is $p(22.5^\circ)$].
- (2) The measurement of detection probability with both polarizers set along the 67.5° axis [that is $p(67.5^\circ)$].
- (3) The measurement of detection probability with the first polarizer set along \mathbf{a}' axis and the second polarizer being removed [that is $p(\mathbf{a}', \infty)$].
- (4) The measurement of detection probability with the first polarizer removed and the second polarizer set along \mathbf{b} axis [that is $p(\infty, \mathbf{b})$].
- (5) The measurement of detection probability with both polarizers removed [that is $p(\infty, \infty)$].

In contrast, the inequality derived in this paper [i.e., inequality (46)] requires the measurements of only two detection probabilities:

- (1) The measurement of detection probability with both polarizers set along the 0° axis [that is $p^{\pm\pm}(0^\circ)$].
- (2) The measurement of detection probability with both polarizers set along the 120° axis [that is $p^{\pm\pm}(120^\circ)$].

Inequality (46) is also experimentally simpler than FC inequality [5] (it should be noted that FC inequality is derived under the assumptions that

³ Note that in an ideal experiment the function $K = 1$. From (20), it can be seen that $E(120^\circ) = -0.5$ and $p^{+-}(0^\circ) = p^{-+}(0^\circ) = 0$. Hence inequality (46) becomes $-1.5 \geq -1$ which violates (41) by a factor of 1.5.

(i) $p(\mathbf{a}', \infty)$ is independent of \mathbf{a}' , (ii) $p(\mathbf{b}', \infty)$ is independent of \mathbf{b}' . These assumptions, however, should be tested experimentally). FC inequality may be written as

$$\frac{p(22.5^\circ) - p(67.5^\circ)}{p(\infty, \infty)} \leq 0.25. \quad (48)$$

The above inequality requires the measurement of at least three detection probabilities:

- (1) The measurement of detection probability with both polarizers set along the 22.5° axis [that is $p(22.5^\circ)$].
- (2) The measurement of detection probability with both polarizers set along the 67.5° axis [that is $p(67.5^\circ)$].
- (3) The measurement of detection probability with both polarizers removed [that is $p(\infty, \infty)$].

In contrast inequality (46) requires the measurements of only two detection probabilities.

VI. Violation of Bell's inequality in phase-momentum and in high-energy experiments

It should be noted that the analysis that led to inequality (46) is not limited to atomic-cascade experiments and can easily be extended to experiments which use phase-momentum [14], or use high energy polarized protons or γ photons [15-16] to test Bell's limit. For example in the experiment by Rarity and Tapster [14], instead of inequality (2) of their paper, the following inequality (i.e., inequality (46) using their notations) may be used to test locality:

$$\frac{3E(120^\circ) - 2\overline{C}_{a_3 b_4}(0^\circ) - 2\overline{C}_{a_4 b_3}(0^\circ)}{K} \geq -1, \quad (49)$$

where $\overline{C}_{a_i b_j}(\phi_a, \phi_b)$ ($i = 3, 4; j = 3, 4$) is the counting rate between detectors D_{a_i} and D_{b_j} with phase angles being set to ϕ_a, ϕ_b (See Fig. 1 of [14]). The following set of orientations $(\phi_a, \phi_b) = (\phi_{b'}, \phi_a) = (\phi_b, \phi_{a'}) = 120^\circ$, and $(\phi_{a'}, \phi_{b'}) = 0^\circ$ leads to the largest violation. Similarly, in high-energy experiments and spin correlation proton-proton scattering experiments [16], inequality (46) can be used to test locality.

VII. Conclusion

In summary, we have derived a correlation inequality [inequality (19)] which can be used to test locality. In case of ideal experiments, this inequality is equivalent to Bell's original inequality of 1965 [1] or CHSH inequality [4]. However, in case of real experiments where polarizers and detectors are not ideal, inequality (19) is a new and distinct inequality and is not equivalent to any of previous inequalities.

We have also demonstrated that the conjunction of Einstein's locality [Eq. (3)] with a supplementary assumption [inequality (34)] leads to validity of inequality (46) that is sometimes grossly violated by quantum mechanics. Inequality (46), which may be called *strong* inequality [17], defines an experiment which can actually be performed with present technology and which does not require the number of emissions N . Inequality (46) requires the measurements of only two detection probabilities (at polarizer angles 0° and 120°), whereas CH or CHSH inequality requires the measurements of five detection probabilities. This result can be of significance for the experimental test of locality where the time during which the source emits particles is usually very limited and it is highly desirable to perform the least number of measurements. Furthermore, QM violates this inequality by a factor of 1.5, whereas it violates CHSH, CH or FC inequality by a factor of $\sqrt{2}$. The larger magnitude of violation can be very useful in actual experiments.

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